

sudoku

3					6		1	
			3	4		7		
	7	1					6	
4					8	9		
	8						4	
		9	2					1
	6					2	7	
		8		7	9			
	4		5					9

<https://www.linurs.org>

Sudoku Solver

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Table of Contents

1. Introduction	1
Copyright	1
Terms	1
Pencil marks	2
Lines	3
2. Solving Techniques	4
Single number techniques	4
Lone single	4
Hidden Single	4
Intersection with block	4
X wing	5
Multiple number techniques	6
Guessing	6
Unique Rectangle	6
Sudoku collection	7
Not yet solved	7
Solved	8
Working	8

Chapter 1. Introduction

A sudoku has 81 cells. In every cell a number between 1 and 9 has to be inserted.

3					6		1	
			3	4		7		
	7	1					6	
4					8	9		
	8						4	
		9	2					1
	6					2	7	
		8		7	9			
	4		5					9

For more see <https://www.learn-sudoku.com>

Copyright

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Terms

A sudoku can be divided in rows:

			3	4		7		
--	--	--	---	---	--	---	--	--

columns:

7
8
6
4

and blocks:

3		
	7	1

When the sudoku is solved every row, column and block must have all numbers from 1 to 9 set in its cells.

Pencil marks

Rows, columns and columns intersect with each other and create therefore restrictions to set the numbers.

Therefore not all numbers can be set into every cell. To solve more complicated sudokus, an overview is required to see what numbers are possible.

This is done by writing pencil marks into the individual cells. A sudoku can therefore be solved by writing all pencil marks into each cell and then erasing the ones that are no more possible until the sudoku is solved. An empty sudoku with pencil marks looks therefore as follows:

1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9

Lines

To remove redundancy in text but also in the programs source code, the term lines has been introduced.

Rows are horizontal lines and columns are vertical lines.

Chapter 2. Solving Techniques

Most sudokus can be solved using the very basic techniques as lone singles and hidden singles. The difficult part is finding hidden singles. Setting pencil marks helps here.

The more advance techniques are unfortunately rarely required.

<https://www.linurs.org/sudoku/index.html> has just the basic techniques implemented and therefore might not be able to solve all sudokus. The sodokus not being able to solve are therefore the interesting ones that are worth to study.

Single number techniques

This group of techniques considers just single number values. This means all pencil marks of other numbers can be ignored.

Lone single

Lone singles are cells where just one number can be inserted. Sometimes they might be not easy to spot. When pencil marks are added then spotting them gets easy.

The following is a cell containing lone singles for 3,6,8,9

8	4	7
2	3	9
5	6	1

Hidden Single

If a number is just possible in one block, row or column then the number can be set and therefore all other pencil mark in this cells can be erased.

The example below has the hidden single 7. The additional pencil marks 5 and 8 make it more difficult to spot.

5 7 8	5 3 8	5 3
2 5 8	4	1
6	2 5	9

Intersection with block

Even when it is not clear where to set a number, some pencil marks can be removed:

Rows and columns intersect with blocks. This intersection contains 3 cells in a line.

If this intersection has the same amount of number as the row or column, then the block can not have the numbers in other cells.

4 5 7	4 5 3 7	5 3	2 7	8	4 2 7	1	9	6
2	1 3 4 7	1 3	6 7 9	4 3 6 9	4 7 9	4 3 4	8	5
6	9	8	1 5	1 3 4 5	1 4 5	2 3 4	2 3	7

The top row has two 3. Both are in the first block (intersect). So one of the two 3 must be a there and all other pencil marks with 3 in the other rows of the first block can be removed.

X wing

When looking to the two columns, the following can be observed:

6	5
2 8	2 8
7	3
2 3 9	6
4	1
2 3 8	2 8
1	7
5	9
2 3	4

The number 8 occurs in the two columns 2 times and on exactly the same rows. The four 8 form a cross or X. It is still not clear where the two 8 are but it can be said that the two 8 must be in

the 4 intersections of the two columns with the two rows. All other 8 pencil marks on the rows not intersection with the two columns can be removed.

Multiple number techniques

xy, pair, triple, quad

Guessing

The aim of solving a sudoku is to use logic and not to guess or try.

Guessing and trying is required when the sudoku has multiple solutions or the skills to solve it are not high enough.

It is a common agreement that sudokus with multiple solutions are invalid. Luckily such sudokus are usually not published or just published by mistake.

If the skills of a human or program is not high enough then guessing is a not very satisfying way to get the solution.

Unique Rectangle

The fact that a sudoku can have just one solution can be used to eliminate pencil marks.

4 ²	8	3	6	5	9	1	7	4 ²
1 ⁶	7	1 ⁶	3	4	2	9	5	8
4 ²	5	9	1	7	8	4 ³ 4 ⁶	4 ³ 4 ⁶	4 ^{2 3} 4 ⁶

In all four corners have the pencil mark 4 and 2. To not run into multiple solutions the pencil marks 2 and 4 can be erased in the lower right cell so just the pencil marks 3 and 6 remain.

<https://www.linurs.org/sudoku/index.html?>

sudoku=...8175...1..23.8....5...2..2..3.8.7..5...2..7.4.8..9..8...9....7.34..5...9178...

The above sudoku has a second unique rectangle, the upper left cell is a 9:

1 6 9	2	1 6
8	3 9	5
7	3 6	4
4	8	3
1 6	7	1 6
2	5	9

Others

<https://www.linurs.org/sudoku/index.html?sudoku=.19..26...73..41.2.621.5..92.631....3842579167.1.4...3645..3..11274..395938521764&pencilmarks=1>

Sudoku collection

The following are difficult sudokus:

Not yet solved

1. <https://www.linurs.org/sudoku/index.html?sudoku=89.....6.18.9...5.47...8..8..1..2..4..9..6..5..3..7.....46.74..6.23.5.6....248>
2. <https://www.linurs.org/sudoku/index.html?sudoku=...6.9..26.3.7...5.1...5.3.436...2.1.9.....7.1.7...569.6.4...5.7...1.3.49..7.3...>
3. <https://www.linurs.org/sudoku/index.html?sudoku=.19..2..82.7...14.34.....2....4..6....7..9.18.....2...2..73..6.8.2.....4.3.1.7..>
4. <https://www.linurs.org/sudoku/index.html?sudoku=...6..5...8.7.3..2...4...91.....2..6.5...8..3..9..47...1..6..9.3.5...4...2...>
5. <https://www.linurs.org/sudoku/index.html?sudoku=8..6...2.4..5..1...7...3.9..4..62.....87...1..5.3...9...1..8..9.4....2..5>
6. <https://www.linurs.org/sudoku/index.html?sudoku=...3425..9.....4..52..9..7.....7..34.271.95..1.....2..3..97..6.....8..9471..>
7. <https://www.linurs.org/sudoku/index.html?sudoku=...9.2..31.9.....4..7..2.8..3.....9...5.6.81..6.....7...14.5.....87...>
8. <https://www.linurs.org/sudoku/index.html?sudoku=.....95...3..7...24.6..8...9..7..4..8.....9...2..1..6...3..6.85...4..7...12.....>
9. <https://www.linurs.org/sudoku/index.html?sudoku=.....42.278....1.1...5..3...4.5.....9...1.....6.7....6..4...3.8....762.52.....>

10. <https://www.linurs.org/sudoku/index.html?>
sudoku=2...69...6..5..324.....8..6.2....9..7..8....5.9..4.....115..3..4...97...2.
11. <https://www.linurs.org/sudoku/index.html?>
sudoku=42...6.716....48..3.6....9...64.....8.7...3..21...4....1.48..1....23..87...1.
12. <https://www.linurs.org/sudoku/index.html?>
sudoku=.27.....2.13...6.5.4...5..6..2....3....9...1..567.....1.8.....6.4...89
13. <https://www.linurs.org/sudoku/index.html?>
sudoku=63.....836.....5.49.23....73.49.....7....9.8....851..4...9.....
14. <https://www.linurs.org/sudoku/index.html?>
sudoku=.....93.7.9..3.61.....4.....1...4...8...7.694.25.....7..32...8.....1.8...7.

Solved

1. <https://www.linurs.org/sudoku/index.html?>
sudoku=8.7..654.1..8.3762....7.81.....8.1962.....85698.....7..4.2..83..4.8..1.8.3..6.4
2. <https://www.linurs.org/sudoku/index.html?>
sudoku=2.....547.8.4.612..5..9.61...2.5...13.7..9..681.5.2.959.....6
3. <https://www.linurs.org/sudoku/index.html?>
sudoku=1837..25.2.9..57.85.7.28..3974..2.3.612....75.....92.4..1....2..12..3...26..9.8.
4. <https://www.linurs.org/sudoku/index.html?>
sudoku=.4.....7.2..63...8...2.8.....2...79..1.....4..39...1.....9.1...3...87..4.2....8.
5. <https://www.linurs.org/sudoku/index.html?>
sudoku=...2...57..9738..4....4..9..9....63.4.6.8.5.2..2....4.723.9.....753...5..2...
6. <https://www.linurs.org/sudoku/index.html?>
sudoku=1.....3...59..2...6.173...81.....68...74.....96...624.3...7..76...5.....4

Working

1. <file:///home/lindegur/Urs/Topics/Sudoku/sudoku/index.html?>
sudoku=312854..996...3.....6.9.3.73....9.....9.2...29638741567..98.....7...9.5294318..&pencilmarks=1

sudoku

<https://www.linurs.org>